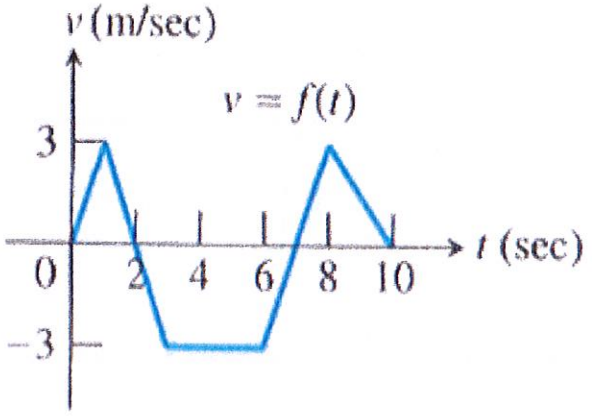
Mr. Michael T. Davis Calculus	Section 3.4 Quiz January 13, 2017

Name: Solutions

1. Particle Motion. The accompanying figure shows the velocity $V = f(t)$ of a particle moving on a coordinate line.



a. When does the particle move forward? Justify your answer.

on $(0,2) \cup (7,10)$ since $V(t) > 0$

b. When does the particle speed up? Justify your answer.

on $(0,1) \cup (2,3) \cup (7,8)$ since $|v(t)|$ is increasing or $|a(t)| > 0$

c. When is the particle's acceleration negative? Justify your answer.

on $(1,3) \cup (8,10)$ since $V'(t) < 0$ or $a(t) < 0$

d. When is the particle's acceleration zero? Justify your answer.

on $(3,6)$ since $V'(t) = 0$ or $a(t) = 0$
since Velocity is constant

e. When does the particle's velocity a constant? Justify your answer.

on $(3,6)$ since $V(t) = -3$ and $V'(t) = 0$ or $a(t) = 0$

2. Projectile Motion. A rocket propelled vertically upward from the surface of the Earth at an initial velocity of $39.2\,m/\text{sec}$ reaches a height of $h(t) = 19t - 4.9t^2$ meters in t seconds.

$h(t) = 39.2t - 4.9t^2$

a. Find the rock's velocity as a function of time.

$V(t) = h'(t) = 39.2 - 9.8t$ m/sec

b. Find the rocket's acceleration as a function of time.

$a(t) = -9.8$ m/sec^2 $\qquad a(t) = V'(t) = h''(t)$

c. What is the rocket's velocity and acceleration at time $t = 6$ sec?

$V(6) = 39.2 - 9.8(6) = 39.2 - 58.8 = -19.6$ m/sec

$a(6) = -9.8$ m/sec^2

d. How long did it take the rocket to reach its highest point? Justify your answer.

$V(t) = 0 \quad 0 = 39.2 - 9.8t \quad 9.8t = 39.2 \quad t = 4$ sec

$V(4) = 0$ the rocket reached its highest point at $t = 4$ since the velocity is zero at $t = 4$ and velocity changes from pos to neg at $t = 4$.

e. How high did the rocket go?

$h(4) = 39.2(4) - 4.9(4)^2 = +78.4$ m

3. Particle Motion. A particle moves along a real number line (left and right) so that its position at any time $t \geq 0$ is given by the function $s(t) = t^3 - 5t^2 + 3t - 2$ where s is measured in meters and t is measured in seconds. Positive velocity implies movement to the right.

a. Determine the particles displacement from $t = 1$ sec and $t = 3$ sec.

$$s(3) - s(1) = 3^3 - 5(3)^2 + 3(3) - 2 - \left(1^3 - 5(1)^2 + 3(1) - 2\right)$$
$$= 27 - 45 + 9 - 2 - (1 - 5 + 3 - 2)$$
$$= -11 - (-3)$$
$$= -8 \text{ meters}$$

b. Determine the average velocity from $t = 1$ sec and $t = 3$ sec.

$$\frac{s(3) - s(1)}{3 - 1} = \frac{-8 \text{ m}}{2 \text{ sec}} = -4 \text{ m/sec}$$

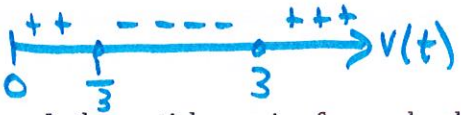
c. Find the instantaneous velocity at any time t.

$$V(t) = 3t^2 - 10t + 3 = (3t - 1)(t - 3)$$

d. At what time(s) does the particle change direction? Justify your answer.

$V(t) = 0 \quad 0 = 3t^2 - 10t + 3 \quad 0 = (3t - 1)(t - 3)$

$t = \frac{1}{3}$ & $t = 3 \quad V(\frac{1}{3}) = 0 \quad V(3) = 0$



The particle changes direction at $t = \frac{1}{3}$ and $t = 3$ since $V(t)$ changes sign

e. Is the particle moving forward or backward at time $t = 4$ sec ? Justify your answer.

$V(4) = 3(4)^2 - 10(4) + 3 = 48 - 40 + 3 = 11$ m/sec

The particle is moving forward since $V(4) = 11$ m/sec which means $V(4) > 0$

f. What is the particle's position when it is the farthest to the left? Justify your answer?

$V(t)$ changes from negative to positive at $t = 3$ sec this means that the particle moves to the left on the interval $(\frac{1}{3}, 3)$ and then moves to the right after $t = 3$ which implies the particle is the farthest to the left at $t = 3$.

$s(3) = 3^3 - 5(3)^2 + 3(3) - 2 = -11$ meters